

Determining V_{ub} from $B^+ \rightarrow D_s^{*+} e^+ e^-$ and $B^+ \rightarrow D^{*+} e^+ e^-$

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(March, 1999)

Abstract

It was recently pointed out that the decays $B^+ \rightarrow D_s^{*+} \gamma$ and $B^+ \rightarrow D^{*+} \gamma$ can be used for an extraction of $|V_{ub}|$. The theory of these decays is poorly understood. It was shown that in a world of almost degenerate b and c -quarks the decay would be computable. The severe difficulties that are encountered in the realistic calculation stem primarily from the very hard photon produced in the two body decay. We point out that in the decays $B^+ \rightarrow D_s^{*+} e^+ e^-$ and $B^+ \rightarrow D^{*+} e^+ e^-$ the photon vertex is soft when the charmed meson is nearly at rest (in the B^+ rest frame). This allows us to compute with some confidence the decay rate in a restricted but interesting kinematic regime. Given enough data the extraction of V_{ub} with reasonably small uncertainties could proceed through an analysis of these exclusive decays much as is done in the determination of V_{cb} .

12.15.Hh, 12.39.Hg, 13.40.Hq, 13.25.Hw

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I. INTRODUCTION

The determination of the Kobayashi-Maskawa (KM) matrix element V_{ub} is one of the central themes of particle physics today. The verification of the KM theory of CP violation in the six quark model requires an independent measurement of the magnitude and phases of the KM matrix elements. The physics community has invested heavily in B -factories, facilities devoted to the measurement of these magnitudes and phases. However, the extraction of V_{ub} from data requires calculation of hadronic transition rates that are marred by the usual host of difficulties introduced by the strong interactions.

For example, the first determination of V_{ub} has been made in the inclusive semileptonic decay of B -mesons. The inclusive decay rate, appropriately smeared over kinematic variables, can be reliably estimated [1]. Since $|V_{cb}|^2 \gg |V_{ub}|^2$, the rate is dominated by the decay into charmed mesons. To isolate the contribution from V_{ub} measurements are made of the decay rate for electron or muon energies that are large enough to exclude contamination from the decay into charm. However, in this very restricted kinematic range the theoretical prediction is highly unreliable, and it is known to be dominated by incalculable non-perturbative effects [2].

A novel procedure offers more hope. If one can reconstruct the neutrino in the semileptonic decay of B mesons only small invariant hadronic masses are required to suppress or eliminate the contribution from decays into charmed particles [3]. The invariant mass spectra are under better theoretical control because the allowed final states are more numerous, so quark-hadron duality is more likely to be valid than in the restricted region of very high electron or muon energies.

In this paper we investigate the extraction of V_{ub} from an exclusive decay mode of charged B -mesons. Exclusive semileptonic decay modes, such as $B \rightarrow \pi \ell \bar{\nu}$ or $B \rightarrow \rho \ell \bar{\nu}$, have proved difficult to study theoretically. The rates are described by non-perturbative form factors which are difficult to estimate. The exclusive decay mode $B^+ \rightarrow D^{*+} e^+ e^-$ depends on non-perturbative dynamics too. But as we will see, under reasonable theoretical assumptions, the non-perturbative dynamics can be extracted from other measurements.

A similar idea was recently proposed in the description of the decays $B^+ \rightarrow D_s^{*+} \gamma$ and $B^+ \rightarrow D^{*+} \gamma$ [4]. There the transition was assumed to be dominated by the lowest poles (B^* and D_s or D) and was given in terms of the amplitude for $B\bar{B}$ mixing and the probability rate for $D^* \rightarrow D\gamma$. The validity of the symmetry arguments used there relied heavily on a theoretical limit, that $m_b - m_c \lesssim \Lambda_{\text{QCD}}$, which is not likely to be a good approximation to reality. This assumption was important in relating the hard photon emission in $B^+ \rightarrow D^{*+} \gamma$ to the soft emission in $D^* \rightarrow D\gamma$. We observe in this paper that for the decay into a lepton pair, rather than a photon, the extended phase space includes a region where the D^* meson is nearly at rest in the B rest-frame, in which the photon is soft. Consequently the unphysical assumption $m_b - m_c \lesssim \Lambda_{\text{QCD}}$ can be dropped.

We make several assumptions and approximations in our calculation of the rates for $B^+ \rightarrow D_s^{*+} e^+ e^-$ and $B^+ \rightarrow D^{*+} e^+ e^-$. If the assumptions prove to hold and the approximations are reliable a determination of V_{ub} through these exclusive modes could proceed in much the same fashion as that of V_{cb} through the exclusive semileptonic mode $B \rightarrow D^* \ell \bar{\nu}$. Theory reliably predicts the rate in the kinematic region in which the D^* is produced at rest in the B rest-frame. Experimentally the rate is measured away from this point and then

extrapolated. The procedure would be entirely analogous for the much rarer, but cleaner, decay we study, $B^+ \rightarrow D^{*+} e^+ e^-$.

II. BASIC ASSUMPTIONS AND SYMMETRIES

Before we present our calculation we discuss in detail the nature of the assumptions and approximations that we use. To lowest order in electromagnetic and weak interactions, the amplitude for the process $B^+ \rightarrow D^{*+} \gamma^*$ is given in terms of

$$\langle D^{*+} | \int d^4x e^{iq \cdot x} T(j_{\text{em}}^\mu(x) \mathcal{O}(0)) | B^+ \rangle. \quad (2.1)$$

Here $\mathcal{O} = \bar{b} \gamma^\nu (1 - \gamma_5) u \bar{c} \gamma^\nu (1 - \gamma_5) d$ is a four quark operator responsible for the weak transition, j_{em}^μ is the electromagnetic current and T stands for time ordering of these operators. For each time ordering one may insert a complete set of states. Our first assumption is that the lowest mass states dominate, giving

$$\begin{aligned} & \int d\Phi \int d^4x e^{iq \cdot x} (\theta(x^0) \langle D^{*+} | j_{\text{em}}^\mu(x) | D^+ \rangle \langle D^+ | \mathcal{O}(0) | B^+ \rangle \\ & + \theta(-x^0) \langle D^{*+} | \mathcal{O}(0) | B^{*+} \rangle \langle B^{*+} | j_{\text{em}}^\mu(x) | B^+ \rangle), \end{aligned} \quad (2.2)$$

where $\int d\Phi$ is the one body phase space. We know of no reliable way to test this assumption. Much of heavy quark chiral perturbation theory [5] relies on analogous assumptions. The effect of excited heavy mesons in loops in chiral perturbation theory has been analyzed and the result seems to be sensitive to the precise values of masses and couplings [6]. This is an area where more research is needed before firm conclusions can be drawn.

Given this assumption, the transition amplitude is calculated in terms of the matrix elements for the weak operator

$$\langle D^+ | \mathcal{O}(0) | B^+ \rangle \quad \text{and} \quad \langle D^{*+} | \mathcal{O}(0) | B^{*+} \rangle \quad (2.3)$$

and the matrix elements of the electromagnetic current,

$$\langle D^{*+} | j_{\text{em}}^\mu(0) | D^+ \rangle \quad \text{and} \quad \langle B^{*+} | j_{\text{em}}^\mu(0) | B^+ \rangle. \quad (2.4)$$

The matrix element of a vector current between a pseudoscalar and a vector meson can be written in terms of a single real form factor, which can be defined as follows

$$\langle \mathbf{p}', \epsilon | j_{\text{em}}^\mu(0) | \mathbf{p} \rangle = i g_X(t) \epsilon^{\mu\nu\lambda\sigma} p_\nu p'_\lambda \epsilon_\sigma^*. \quad (2.5)$$

Here $X = B$ or D identifies the one particle states, ϵ is the polarization of the vector meson, and the time component of the four vectors p and p' is the energy of the state, $p_0 = \sqrt{M^2 + |\mathbf{p}|^2}$. The Lorentz invariant form factor g_X is a function only of the invariant momentum transfer $t = (p - p')^2$. Heavy quark flavor symmetry [7] can be used to relate g_B and g_D when the argument is $t \approx 0$. Moreover, $g_X(0) = \mu_X$, where μ_X is the magnetic moment defined in Ref. [8] determines the radiative decay rate of the D^* meson [8],

$$\Gamma(D^* \rightarrow D\gamma) = \frac{\alpha}{3} |\mu_X|^2 \left(\frac{M_{D^*}^2 - M_D^2}{2M_{D^*}} \right)^3. \quad (2.6)$$

The heavy quark contribution to the magnetic moment of the meson is suppressed in the large mass limit, so $\mu_X = \mu Q_q$, where Q_q is the charge of the light quark or anti-quark in the heavy meson. Therefore $\mu_{D^+} \approx \frac{1}{2}\mu_{B^+}$.

The functional form of the form factor $g(t)$ is not known. Clearly the dimensionful scale that determines its behavior is on the order of the mass of the lightest vector meson, the ρ -meson. Now, in the rest-frame of the decaying B^+ meson, $\mathbf{p} = 0$, and $t = (p'_0 - M)^2 - |\mathbf{p}'|^2 = (M' - M)^2 - 2M(p'_0 - M')$. So at $\mathbf{p}' = 0$ the momentum transfer variable is small and positive, $t \approx (50 \text{ MeV})^2$ for B mesons and $t \approx (150 \text{ MeV})^2$ for D mesons. As $|\mathbf{p}'|$ increases t goes through zero and turns negative. Therefore there is a region of phase space, close to the point of zero recoil of the D^* meson, for which the matrix elements in (2.4) are known directly from experiment or via heavy quark symmetry.

It is not obvious how the form factors $g(t)$ should be incorporated in the calculation. Using the matrix element given in terms of a form factor in Eq. (2.5) in the expression for the amplitude in Eq. (2.2) gives non-covariant results. For a constant or linear form factor a covariant result is recovered if the “Z” graphs are included (ie, for $x^0 < 0$ a graph with an intermediate state of the three particles D^- , D^{*+} and B^+). But covariance is lost for more general form factors. Although the “Z” graphs are needed to recover covariance, the behavior of the form factor suggests that they are negligible in practice. In what follows we will compute in the rest frame of the decaying B^+ meson and neglect “Z” graphs. This is a good approximation at least when the resulting D^* meson is at rest or recoiling slowly. The question of how to incorporate the form factor effects consistently is an open one and we hope to return to it in the future.

Turning our attention to the weak matrix elements in (2.3) we notice immediately the close resemblance of these to the matrix elements in $B\bar{B}$ mixing,

$$\langle \bar{B} | \mathcal{O}_B | B \rangle = \frac{8}{3} f_B^2 M_B^2 B_B. \quad (2.7)$$

Here $\mathcal{O}_B = \bar{b}\gamma^\nu(1 - \gamma_5)d\bar{b}\gamma_\nu(1 - \gamma_5)d$ and the right hand side parameterizes the matrix element in terms of B_B and the mass M_B and decay constant f_B of the B -meson. In both cases a heavy quark and a light quark are annihilated at a local vertex where a heavy anti-quark and a light quark are created. In $B\bar{B}$ mixing the heavy anti-quark is produced at rest. In $B^+ \rightarrow D^*\ell^-\ell^+$ we already singled out the case of zero recoil as the one where we can compute the electromagnetic transition. The crudest approximation, vacuum insertion, works surprisingly well in the case of B -mesons, at least as indicated by calculations in quenched lattice QCD. Vacuum insertion gives

$$\langle D^+ | \mathcal{O}(0) | B^+ \rangle = f_B f_D M_B M_D, \quad (2.8)$$

and the analogous result for the vector mesons.

One may try a more sophisticated attempt at predicting this matrix element, using symmetries to relate it to the measured matrix element for $B\bar{B}$ mixing. While heavy quark symmetries do allow us to exchange the heavy b anti-quark for a heavy c anti-quark, isospin (or

$SU(3)$, for the D_s^*) which is needed to exchange the light quarks does not give a relation between the matrix elements. The operator \mathcal{O} transforms in the reducible $\mathbf{3} \oplus \mathbf{1}$ representation of isospin ($\mathbf{6} \oplus \bar{\mathbf{3}}$ representation of $SU(3)$), but the corresponding operator for $B\bar{B}$ mixing transforms in the irreducible $\mathbf{3}$ of isospin ($\mathbf{6}$ of $SU(3)$). We make the plausible assumption that the matrix element of the color octet operator $\mathcal{O}_8 = \bar{b}T^a\gamma^\nu(1 - \gamma_5)u\bar{c}T^a\gamma_\nu(1 - \gamma_5)d$ is negligible. Indeed, if one could insert a complete set of (gauge invariant) states between the currents in \mathcal{O}_8 , gauge invariance would immediately imply that the matrix element vanishes. Inserting a complete set of states seems less drastic than assuming saturation by the vacuum alone. The assumption that \mathcal{O}_8 is negligible leads to relations between the matrix elements of the $\mathbf{3}$ and $\mathbf{1}$ components of \mathcal{O} . With $P_\pm \equiv 1 \pm \gamma_5$, define

$$\mathcal{O}_\pm = \bar{b}\gamma^\nu P_- u \bar{c}\gamma^\nu P_- d \pm \bar{b}\gamma^\nu P_- d \bar{c}\gamma^\nu P_- u \quad (2.9)$$

then \mathcal{O}_+ and \mathcal{O}_- transform in the $\mathbf{3}$ and $\mathbf{1}$ representations of isospin and we have the change of basis relations:

$$\begin{pmatrix} \mathcal{O} \\ \mathcal{O}_8 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} \mathcal{O}_+ \\ \mathcal{O}_- \end{pmatrix} \quad (2.10)$$

The vanishing of the matrix elements of \mathcal{O}_8 implies that between B^+ and D^{*+} states $\langle \mathcal{O}_- \rangle = \frac{1}{2}\langle \mathcal{O}_+ \rangle$ and therefore $\langle \mathcal{O} \rangle = \frac{3}{4}\langle \mathcal{O}_+ \rangle$.

In order to relate the matrix elements of \mathcal{O}_B and \mathcal{O} we also need to make use of heavy quark symmetries. The first step is to write operators in a heavy quark effective theory (HQET) $\tilde{\mathcal{O}}_B$ and $\tilde{\mathcal{O}}$ corresponding, respectively, to the operators \mathcal{O}_B and \mathcal{O} in the original theory. If $h_{\bar{Q}}$ is an HQET field that destroys a heavy quark of four-velocity v , and h_Q^\dagger an HQET field that creates a heavy antiquark with the same velocity v then

$$\mathcal{O}_B \rightarrow \tilde{\mathcal{O}}_B = 2 h_b^T \gamma^0 \gamma^\nu P_- d \bar{h}_b \gamma_\nu P_- d. \quad (2.11)$$

The superscript T stands for the transpose. The factor of two arises from the two ways of obtaining a heavy quark creation and corresponding anti-quark annihilation operators. Similarly

$$\begin{aligned} \mathcal{O}(x) \rightarrow \tilde{\mathcal{O}}(x) &= e^{-i(m_b - m_c)v \cdot x} \\ &\times h_b^T(x) \gamma^0 \gamma^\nu P_- u(x) \bar{h}_c(x) \gamma_\nu P_- d(x), \end{aligned} \quad (2.12)$$

and analogous correspondences for \mathcal{O}_8 and \mathcal{O}_\pm . In the HQET there is a $SU(2) \times SU(2)$ flavor symmetry that rotates $b \leftrightarrow c$ and $\bar{b} \leftrightarrow \bar{c}$ independently. The symmetry is explicit between states with mass independent normalization. Using this together with suppression of \mathcal{O}_8 and isospin symmetry we have

$$\frac{\langle D^+ | \tilde{\mathcal{O}} | B^+ \rangle}{\sqrt{M_B M_D}} = \frac{3}{4} \frac{\langle D^+ | \tilde{\mathcal{O}}_+ | B^+ \rangle}{\sqrt{M_B M_D}} = \frac{1}{2} \frac{3}{4} \frac{\langle \bar{B}^0 | \tilde{\mathcal{O}}_B | B^0 \rangle}{M_B}. \quad (2.13)$$

The factor of 1/2 in the last step is a Clebsch-Gordan coefficient. Note that the factor of 2 appearing in Eq. (2.11) was retained in the definition of $\tilde{\mathcal{O}}_B$ so that $\tilde{\mathcal{O}}_B$ and $\tilde{\mathcal{O}}_+$ have the

same normalization in the triplet tensor operator. In terms of B_B defined in Eq. (2.7) we finally obtain for states with vanishing momentum

$$\langle D^+ | \mathcal{O}(x) | B^+ \rangle = e^{-i(m_b - m_c)t} \sqrt{\frac{M_D}{M_B}} f_B^2 M_B^2 B_B, \quad (2.14)$$

or, using $f_B^2 M_B = f_D^2 M_D$ from heavy quark symmetry, and evaluating at the origin

$$\langle D^+ | \mathcal{O}(0) | B^+ \rangle = f_B f_D M_B M_D B_B. \quad (2.15)$$

Comparing with Eq. (2.8) we see that the symmetry argument and the octet suppression assumption give the result that the B correction factor for $B^+ \rightarrow D^+$ is precisely B_B , the correction factor for $B\bar{B}$ mixing.

The mixing between B^{*+} and D^{*+} in (2.3) is fixed by heavy quark spin symmetry [9] to have the same magnitude but opposite sign than Eq. (2.15). As a consequence of this sign the contributions to the rate from the two time orderings in Eq. (2.2) add. However, in the decay to a pseudoscalar, $B^+ \rightarrow D^+ e^+ e^-$, the two time orderings tend to cancel each other. Moreover, the electromagnetic form factors in $B^+ \rightarrow D^+ e^+ e^-$ are unity at $t = 0$, so the cancellation is rather good. For this reason we have not analyzed this process further.

III. RESULTS AND DISCUSSION

The effective Hamiltonian for the weak transition is modified by short distance QCD corrections,

$$\mathcal{H}'_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* (c(\mu/M_W) \mathcal{O} + c_8(\mu/M_W) \mathcal{O}_8). \quad (3.1)$$

The short distance coefficients c and c_8 are chosen so that their dependence on the renormalization point μ cancels the μ -dependence of operators, in such a way that matrix elements of the effective Hamiltonian are μ -independent. Resuming the leading logs, they are given at $\mu = m_b$ in terms of $x = (\alpha_s(m_b)/\alpha_s(M_W))^{6/23}$ by $c = \frac{1}{3}x^2 + \frac{2}{3}x^{-1}$ and $c_8 = x^{-1} - x^2$; numerically, with $\alpha_s(m_b)/\alpha_s(M_W) \approx 1.9$ one has $c \approx 1.0$ and $c_8 \approx -0.6$.

The calculation is now straightforward. The transition amplitude for $B^+ \rightarrow D^{*+} e^+ e^-$ is given by the vector in Eq. (2.2) contracted with the leptonic current and multiplied by the photon propagator and coupling constants. The hadronic matrix elements in Eq. (2.2) are computed as described in the previous section.

There is one subtlety. For the first time ordering in Eq. (2.2) the energy denominator is $q_0 + E_{D^{*+}} - E_D = M_B - E_D$, where $E_D = \sqrt{E_B^2 - M_B^2 + M_D^2}$ is the energy of the intermediate D -meson. For the second time ordering the energy denominator is $q_0 - E_B + E_{B^*} = -E_{D^*} + E_{B^*}$, where the intermediate B^* energy is $E_{B^*} = \sqrt{E_{D^*}^2 - M_{D^*}^2 + M_{B^*}^2}$. In the rest frame of the B -meson and at zero D^* recoil, these denominators are $M_B - M_D$ and $-(M_B - M_D) + O(1/M)$. For non-zero D^* recoil, the denominators are different. Therefore our results do not depend on the form factors through the simple combination $g_B(t_B) + g_D(t_D)$, except at the point of zero recoil.

The differential decay rate is best written in terms of two functions that contain the kinematic dependence. The dependence on the vector current form factors is in

$$\mathcal{G}(q^2) = N_{\mathcal{G}}^{-1} \left(g_D(t_D) + \frac{M_B(M_B - M_D)}{E_{B^*}(E_{B^*} - E_{D^*})} g_B(t_B) \right), \quad (3.2)$$

where $t_X = (M_X - M_{X^*})^2 - 2M_X(E_{X^*} - M_{X^*})$ and $N_{\mathcal{G}}$ is a normalization factor defined so that $\mathcal{G} = 1$ at $q^2 = q_{\text{max}}^2 = (M_B - M_{D^*})^2$ and is approximately

$$N_{\mathcal{G}} \approx \mu_D + \mu_B \approx 3\mu_D. \quad (3.3)$$

The second dimensionless function is a combination of amplitude and phase space dependence. It vanishes at the upper end-point as $(q_{\text{max}}^2 - q^2)^{3/2}$. It is given by

$$\begin{aligned} \mathcal{F}(q^2) &= \frac{\sqrt{1 - 4m_e^2/q^2}(1 + 2m_e^2/q^2)}{q^2 M_B^4} \\ &\times (q^2 - (M_B + M_D)^2)^{3/2} (q^2 - (M_B - M_D)^2)^{3/2}. \end{aligned} \quad (3.4)$$

In terms of these functions we finally obtain

$$\frac{d\Gamma}{dq^2} = \frac{3\alpha^2}{64\pi} G_F^2 |V_{ub}V_{cd}|^2 c^2 \frac{f_B^4 M_B^2 M_D B_B^2}{(M_B - M_D)^2} \mu_D^2 \mathcal{G}^2(q^2) \mathcal{F}(q^2). \quad (3.5)$$

Using Eq. (2.6) the differential branching fraction for $B^+ \rightarrow D^{*+}e^+e^-$ is, numerically,

$$\begin{aligned} \tau_B M_B^2 \frac{d\Gamma}{dq^2} &= 1.3 \times 10^{-11} \left(\frac{|V_{ub}V_{cd}|}{8 \times 10^{-4}} \right)^2 \left(\frac{f_B \sqrt{B_B}}{170 \text{ MeV}} \right)^4 \\ &\times \left(\frac{\Gamma(D^* \rightarrow D\gamma)}{4.2 \text{ KeV}} \right) \mathcal{G}^2(q^2) \mathcal{F}(q^2), \end{aligned} \quad (3.6)$$

and about a factor of 16 bigger for $B^+ \rightarrow D_s^{*+}e^+e^-$. The integrated branching fraction, restricted to $q^2 > 0.1 \text{ GeV}^2$, is 6.3×10^{-10} assuming \mathcal{G} is constant, but for \mathcal{G} given by a single ρ -pole, $\mathcal{G} = m_\rho^2/(m_\rho^2 + q^2 - q_{\text{max}}^2)$, it is reduced to 2.2×10^{-12} (and, again, a factor of 16 bigger for $B^+ \rightarrow D_s^{*+}e^+e^-$).

IV. CONCLUSIONS

We have calculated the rates for $B^+ \rightarrow D^{*+}e^+e^-$ and $B^+ \rightarrow D_s^{*+}e^+e^-$. Our primary motivation is the observation that at the kinematic point where the $D_{(s)}^*$ meson is recoiling slowly in the B rest frame the amplitude can be computed in terms of other observables, like the width of the D^* and the $B - \bar{B}$ mixing rate. While this observation opens up exciting possibilities of computation and of the determination of V_{ub} , we leave many loose ends. The computation makes several assumptions which need to be further tested. Among these the assumption that the rate is dominated by the lowest mass resonance is most suspect. Also, we leave unanswered a fundamental question: how can one covariantly incorporate the electromagnetic form factors $g(t)$, which are here assumed to be known, into the calculation?

We are not discouraged by the seemingly small branching fraction obtained. The theoretical questions and challenges raised (see previous paragraph) are worth pursuing in their own right. Their resolution may guide us to a better method for a determination of V_{ub} in exclusive decays.

Acknowledgments This work is supported by the Department of Energy under contract No. DOE-FG03-97ER40546.

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